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Critical behaviour of nematic liquid crystals in oblique magnetic fields

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The static critical behaviour of a bulk nematic liquid crystal sample in an oblique magnetic field is analysed. When a magnetic field is applied at a suitable angle α with respect to the initially homogeneous nematic director, a spatially inhomogeneous director pattern can be formed. The transition to the deformed state and the formation of walls between the domains resulting from the two equally stable configurations above the transition are studied. The width of the walls is found to diverge at the transition. The critical exponents corresponding to the transition and wall formation are shown to be characteristic of a mean field second order phase transition.

1. Introduction

The study of the stability of the nematic director under external fields is usually a difficult task, due to the complexity of the phenomena arising from the highly non-linear system of equations which usually describe its behaviour. Although this system is not a gradient system, and the dissipative part usually plays an important role in its dynamics, the existence of a free energy of the system and its study are sufficient to account for an understanding of the static phenomena. The static study of the external field-induced instabilities in nematic liquid crystals is usually carried out using minimization techniques involving hard to solve differential equations [1, 2]. Simpler techniques can be of great help in dealing with problems that could only otherwise be solved using drastic approximations. Here, one such technique is applied, involving the use of an ansatz for the director field in the frame of continuum theory and then the study of the stability of that field using the general methods of stability theory [3].

For oblique magnetic fields making an angle α with respect to the initially aligned, unperturbed director, the existence of a critical angle α_c for the development of periodic distortions in the director has been explained theoretically for a bulk nematic monodomain, using a two-dimensional director field static analysis, with a simple ansatz for the tilt angle [4]. In reference [3] a generalization of the ansatz carried out in reference [4] was implemented by considering a three-dimensional director field ansatz instead of a two-dimensional one as in reference [4]. It was shown that for nematics with positive anisotropy of the magnetic susceptibility χ_a this leads to the same results as the two-dimensional director field, while for $\chi_a < 0$ this is only true for a pure bend distortion mode. For oblique wavevectors, and depending on the relative values of the ratios of the elastic constants, the azimuthal component of the distortion can be excited in the case $\chi_a < 0$.

The goal of this paper is to study the critical behaviour of the nematic director field in the oblique magnetic field geometry, following the (static) stability analysis approach. For simplicity, only nematics with positive anisotropy of the magnetic susceptibility and bulk samples with free boundaries are considered. In the first part of this work, the transition of the director from a homogeneous to a periodic deformed state is studied using a two-dimensional nematic field ansatz, but with a generalization of the function of [4] as the perturbation. Next, the formation of two equally stable configurations above the transition and the walls between the resulting domains are studied. Finally, in order to characterize the transition, the corresponding critical exponents are obtained.

2. Static stability analysis

When the magnetic field is applied at a suitable angle with the initial homogeneous director field, the reorientation of the director results in a transient periodic pattern with backflow. This pattern has been observed experimentally by optical and NMR techniques (see [4] and references therein). In this dynamic process the state with a periodic director field is a metastable state. In the static stability analysis approach the transition to the periodic director field will correspond to a local change of stability of the free energy of the system.

Consider a bulk aligned monodomain of a nematic with positive anisotropy of the magnetic susceptibility. A magnetic field $\mathbf{H}=(H\sin\alpha, 0, H\cos\alpha)$ is applied at an angle α with respect to the initial homogeneous director $\mathbf{n}_0=(0,0,1)$, as shown in figure 1. To study the stability of the director field with respect to the development of a deformation, a two-dimensional perturbed director field will be used:

$$\mathbf{n}=(\sin\theta, 0, \cos\theta). \tag{1}$$

Since this work concerns nematic systems with $\chi_a>0$ and, according to the three-dimensional director static stability analysis of [3], the azimuthal perturbation ϕ should only be excited in the $\chi_a<0$ case.

In order to study the transition to the distorted director field, a generalization of the ansatz is used for the perturbation to the tilt angle θ of [4], in the form of a Fourier series, starting with a (one-dimensional) bend distortion:

$$\theta(z)=\sum_{j=1}^{\infty}\theta_{0j}\sin(j\Omega), \qquad \Omega=q_z z. \tag{2}$$

When all terms with $j>1$ are put to zero in equation (2), the ansatz of [4] is recovered. With the ansatz (2) the corresponding gain in distortion free energy density can be computed with the help of the Oseen–Zöcher–Frank continuum theory [5]

$$f_d=\frac{1}{2}K_1[\nabla\cdot\mathbf{n}]^2+\frac{1}{2}K_2[\mathbf{n}\cdot(\nabla\times\mathbf{n})]^2 + \frac{1}{2}K_3[\mathbf{n}\times(\nabla\times\mathbf{n})]^2 \tag{3}$$

where the elastic constants K_i, $i=1,2,3$ correspond to the three independent elastic modes of splay, twist and bend, respectively. The surface-like terms involving the elastic constants k_{24} and k_{13} are not considered here since they should not play a role for a two-dimensional director field as (1) [3].

To obtain the total free energy density $f(\theta_{01},\ldots,\theta_{0j},\ldots,\Omega)$ of the system under study, one must add to the elastic term (3) a magnetic term

$$f_{\text{mag}}=-\frac{1}{2}\chi_a(\mathbf{n}\cdot\mathbf{H})^2. \tag{4}$$

For a bulk sample, the interesting quantity is the mean free energy density per wavelength of the distortion:

$$F(\theta_{01},\ldots,\theta_{0j},\ldots)=\frac{1}{2\pi}\int_0^{2\pi}f(\theta_{01},\ldots,\theta_{0j},\ldots,\Omega)\,d\Omega. \tag{5}$$

The amplitudes $\theta_{0j}, j=1,2,\ldots$, play the roles of order parameters of the system: a state with $(\theta_{0j}=0, j=1,2,\ldots)$ represents the unperturbed homogeneous director, while a state with one or more of the amplitudes $\theta_{0j}\neq 0$ represents a distorted director. The goal is to investigate the stability of the potential (5) with respect to the onset of a distortion. This means studying the stability of the potential at the origin in the order parameter space as a function of the control parameters: the anisotropy of the magnetic susceptibility χ_a, the elastic constants K_i, $i=1,2,3$ and the external parameters H and α. The wavevector $\mathbf{q}$ is an internal parameter of the system. In a static analysis, its selection is dictated by the boundary conditions. Here it will be considered as a free parameter, since we are dealing with no boundary conditions. A stability analysis based on specific boundary conditions will be considered elsewhere [6].

The qualitative properties of a potential at a point are governed by the lowest degree terms of its Taylor series expansion about that point. The expansion up to second order around the origin of the potential (5) can be written in a dimensionless form as

$$\begin{aligned}\Phi(\theta_{01},\ldots,\theta_{0j},\ldots)&\equiv 4F(\theta_{01},\ldots,\theta_{0j},\ldots)/\chi_a H^2\\ &=\Phi_0+\sum_{j=1}^{\infty}a_j\theta_{0j}^2+O(\theta_{0j}^4)\end{aligned} \tag{6}$$

with

$$\Phi_0=-(1+\cos 2\alpha) \tag{7}$$

and

$$a_j=j^2\varepsilon+\cos 2\alpha \tag{8}$$

where

$$\varepsilon=\xi_3^2 q_z^2 \tag{9}$$

where ξ_3 is the bend magnetic coherence length: $\xi_3^2=K_3/\chi_a H^2$.

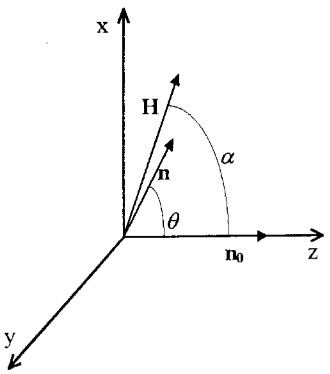

Figure 1. Definition of the sample geometry. Homogeneous initial orientation of the director: $\mathbf{n}_0=(0,0,1)$. Magnetic field: $\mathbf{H}=(H\sin\alpha, 0, H\cos\alpha)$. Perturbed director: $\mathbf{n}=(\sin\theta, 0, \cos\theta)$.

The state of deformation of the director is described by the equilibrium points of the system, which are determined by the condition

$$\text{grad } \Phi = 0 \tag{10}$$

where the derivatives are taken with respect to the order parameters θ_{0j}. The origin is a trivial solution of equation (10). The stability properties of the equilibrium points may be determined from the eigenvalues of the static stability matrix [7]:

$$\Phi_{kl} = \partial^2 \Phi / \partial k \partial l \tag{11}$$

where $k \equiv \theta_{0k}$ and $l \equiv \theta_{0l}$, $k, l = 1, 2, \ldots$. The stability matrix (11), with (6), is diagonal, with eigenvalues

$$\lambda_j = a_j, \qquad j = 1, 2, \ldots . \tag{12}$$

If all eigenvalues are positive, the equilibrium point is a local minimum. When all are negative, the point is a local maximum. In the other cases, the point has a saddle shape. Only in the first case is the origin then (locally) stable. The origin is a *critical point* (where there is change of stability or *bifurcation*) when the control parameters are such that one or more eigenvalues assume the value zero, which implies:

$$\det[\Phi_{kl}]_{k=0, l=0} = 0. \tag{13}$$

This means that the critical points of the system are at

$$a_j = 0, \qquad j = 1, 2 \ldots . \tag{14}$$

Solving equation (14) in order to obtain the angle α gives the critical angle α_c (for which bifurcation occurs) as a function of the control parameters, for each of the Fourier components of the function $\theta(z)$

$$\cos 2\alpha_c = -j^2 \varepsilon, \qquad j = 1, 2, \ldots \tag{15}$$

where ε is given by equation (9). Plotting the critical angle as a function of ε one gets phase diagrams as shown in figure 2. The lower curve for α_c corresponds to $j = 1$. The figure shows that for given ε the harmonics of order greater than one can only be excited if the first one is also excited. Since there are no possible constraints that can force the director to stay in a non-deformed unstable state with $\theta_{01} = 0$, this means that the first critical point lies at $a_1 = 0$. We can then conclude that θ_{01} is the order parameter of the system, which will simply be called θ_0 in what follows. The results for the critical angle of [4] for a bend mode are then unchanged by the addition of higher harmonics in the ansatz for the perturbed director.

These results are readily generalized for distortions in more than one dimension. If $\mathbf{q}$ is a three-dimensional wavevector, we get from the first critical point

$$\cos 2\alpha_c = -\varepsilon \tag{16}$$

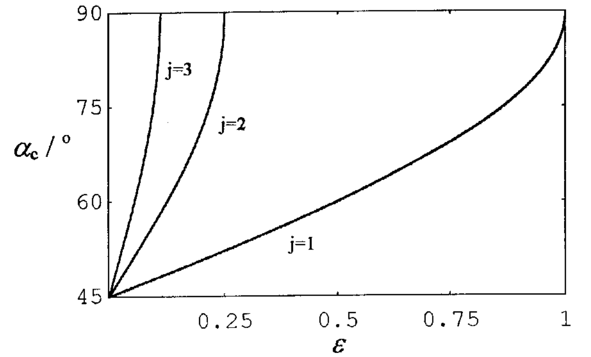

Figure 2. Phase diagrams from equation (15) for the first three harmonics. This shows that, for $\chi_a > 0$, $\alpha_c \in [45°, 90°]$. The maximum value of ε for each harmonic up to which a solution exists is $\varepsilon_{max}(j) = 1/j^2$. The lower curve defines the line of transition points (see text).

with

$$\varepsilon = \xi_1^2 q_x^2 + \xi_2^2 q_y^2 + \xi_3^2 q_z^2 \tag{17}$$

where the ξ_i, $i = 1, 2, 3$, are the splay, twist and bend magnetic coherence lengths respectively: $\xi_i^2 = K_i / \chi_a H^2$.

3. Walls

Due to the director invariance $\mathbf{n} = -\mathbf{n}$, a uniform nematic with $\chi_a > 0$ will align parallel or, equivalently, anti-parallel with respect to the direction of an applied magnetic field. In the oblique magnetic field geometry, above the transition at $\alpha = \alpha_c$, two equally stable configurations will then be allowed. The pattern and the energy of the walls between the corresponding domains can be obtained using a similar method to that used by Brochard in the (static) study of the formation of walls in the Fréedericks transition [8]. Brochard walls and Helfrich inversion walls are two different kinds of defects that can arise in magnetic fields [5]. A Helfrich splay–bend wall, following a 90° rotation of the sample, was studied in reference [4]. Here, interest is in the formation of walls at the transition. For simplicity, only walls following the development of two-dimensional instabilities are studied here.

In the first place, the possibility of the formation of a twist wall in the presence of a splay–bend instability (in the plane perpendicular to the twist) is investigated, and therefore the following director is considered:

$$\theta(\mathbf{r}) = \theta_0(y) \sin \Omega, \qquad \Omega = q_x x + q_z z. \tag{18}$$

The goal is to determine the angle $\theta_0(y)$ when going continuously from $-\theta_0$ at $y = -\infty$ to $+\theta_0$ at $y = +\infty$. The corresponding distortion free energy density, averaged over a wavelength of the distortion, after performing similar calculations to those described in reference [4—Appendix A1], is given by

$$F(y)=\frac{1}{8}q_z^2\theta_0^2\left[K_1+K_3+(K_3-K_1)\frac{J_1(2\theta_0)}{\theta_0}\right]+\frac{1}{8}q_x^2\theta_0^2\left[K_1+K_3-(K_3-K_1)\frac{J_1(2\theta_0)}{\theta_0}\right]+\frac{1}{4}K_2\left(\frac{\mathrm{d}\theta_0}{\mathrm{d}y}\right)^2-\frac{1}{4}\chi_a H^2[1+\cos(2\alpha)J_0(2\theta_0)] \quad (19)$$

where the J_i, $i=0,1$, are Bessel functions of the first kind. It is more convenient to work with an adimensional potential, which can be taken as

$$\Phi(y)\equiv\frac{4F(y)}{\chi_a H^2}=\xi_2^2\left(\frac{\mathrm{d}\theta_0}{\mathrm{d}y}\right)^2+\frac{1}{2}\xi_1^2\theta_0^2[q_z^2 g_b(\theta_0)+q_x^2 g_s(\theta_0)]-[1+\cos(2\alpha)J_0(2\theta_0)] \quad (20)$$

where

$$g_b(\theta_0)=\rho_K+1+(\rho_K-1)\frac{J_1(2\theta_0)}{\theta_0} \quad (21\,a)$$

and

$$g_s(\theta_0)=\rho_K+1-(\rho_K-1)\frac{J_1(2\theta_0)}{\theta_0} \quad (21\,b)$$

with $\rho_K=K_3/K_1$.

The equilibrium configuration of the director is obtained by minimization of Φ with respect to $\theta_0(y)$. Using results of reference [4—Appendix A2], one gets the following Euler–Lagrange equation:

$$\xi_2^2\frac{\mathrm{d}^2\theta_0}{\mathrm{d}y^2}=\frac{1}{2}\xi_1^2\theta_0[q_x^2 h_s(\theta_0)+q_z^2 h_b(\theta_0)]+\cos(2\alpha)J_1(2\theta_0) \quad (22)$$

with

$$h_b(\theta_0)=\rho_K+1+(\rho_K-1)J_0(2\theta_0) \quad (23\,a)$$

and

$$h_s(\theta_0)=\rho_K+1-(\rho_K-1)J_0(2\theta_0). \quad (23\,b)$$

Not too far from the transition, we can keep terms only up to the third order in θ_0 in the rhs of equation (22), giving

$$2\xi_2^2\frac{\mathrm{d}^2\theta_0}{\mathrm{d}y^2}=a\theta_0+b\theta_0^3 \quad (24)$$

with

$$a=2(\xi_1^2 q_x^2+\xi_3^2 q_z^2+\cos 2\alpha) \quad (25\,a)$$

or, using equation (16) with $q_y=0$

$$a=2(\cos 2\alpha-\cos 2\alpha_c) \quad (25\,b)$$

and

$$b=(q_z^2-q_x^2)(\xi_1^2-\xi_3^2)-\cos 2\alpha. \quad (26)$$

A first integration of equation (24) gives

$$\frac{\xi_2^2}{b}\left(\frac{\mathrm{d}\theta_0}{\mathrm{d}y}\right)^2=\left(\frac{\theta_0^2-\theta_0^2}{2}\right)^2 \quad (27)$$

where

$$\theta_0=(-a/b)^{1/2} \qquad (a<0) \quad (28)$$

is the solution for $y=\pm\infty$.

θ_0 is also the equilibrium value of the order parameter θ_0 in the periodic deformed state, as will be shown in the following steps. In order to describe the state of the director for $a<0$ ($\alpha>\alpha_c$), it is necessary to look at the terms of higher than second degree in the single variable $\theta_0\equiv\theta_{01}$ in the expansion (6). All other variables θ_{0j}, $j=2,3,\ldots$ may be neglected. Keeping terms up to fourth order in θ_o [at the transition, at $a=0$, b is positive for known materials, and consequently subsequent terms in the expansion (6), for the single variable θ_0, cannot alter the critical behaviour of the system at the origin], one gets the potential

$$\Phi=\Phi_0+a\theta_0^2/2+b\theta_0^4/4 \quad (29)$$

with Φ_0 given by equation (7), from where it follows trivially that θ_0 given by (28) is the non-zero solution of the equation $\mathrm{d}\Phi/\mathrm{d}\theta_0=0$ (when $a<0$).

Integration of equation (27) yields the result

$$\theta_0(y)=\theta_0\,\mathrm{th}\left[\frac{(-a)^{1/2}}{2\xi_2}y\right], \qquad (a<0) \quad (30)$$

corresponding to a twist wall of width l_t of order

$$l_t\approx 2\xi_2/(-a)^{1/2}. \quad (31)$$

This result shows that $l_t\to\infty$ when $\alpha\to\alpha_c$. Comment is made in the next section on this divergence of the wall width, which is similar to that found with Brochard walls at the Fréedericks transition [5, 8]. The energy stored per unit surface of the twist wall can be easily computed using equation (27):

$$W_t=\frac{1}{2}K_2\int_{-\infty}^{\infty}\left(\frac{\mathrm{d}\theta_0}{\mathrm{d}y}\right)^2\mathrm{d}y=\frac{1}{2}K_2\int_{-\theta_0}^{\theta_0}\frac{\mathrm{d}\theta_0}{\mathrm{d}y}\mathrm{d}\theta_0=\frac{1}{3}\frac{K_2}{\xi_2}\frac{(-a)^{3/2}}{b} \qquad (a<0) \quad (32)$$

which is positive, since $b>0$ for $a\leqslant 0$.

Next the possibility of the formation of a splay–bend wall is studied. To search for such a conformation of the director, the following ansatz is chosen

$$\theta(\mathbf{r})=\theta_0(x)\sin\Omega, \qquad \Omega=q_y y+q_z z \quad (33)$$

and the angle $\theta_0(x)$ is sought when going continuously from $-\theta_0$ at $x=-\infty$ to $+\theta_0$ at $x=+\infty$. The corresponding free energy, averaged over a wavelength and written in dimensionless form, gives the following potential

$$\Phi(x)=\xi_2^2 q_y^2\theta_0^2+\frac{1}{2}\xi_1^2\left[q_z^2\theta_0^2 g_{\mathrm{b}}(\theta_0)+\left(\frac{\mathrm{d}\theta_0}{\mathrm{d}x}\right)^2 g_{\mathrm{s}}(\theta_0)\right] - [1+\cos(2\alpha)J_0(2\theta_0)]. \tag{34}$$

Proceeding as before, the corresponding Euler–Lagrange equation, keeping terms up to θ_0^3, is

$$\xi_1^2\left\{2\frac{\mathrm{d}^2\theta_0}{\mathrm{d}x^2}+\left(\frac{K_3}{K_1}-1\right)\left[\frac{\theta_0^2}{2}\frac{\mathrm{d}^2\theta_0}{\mathrm{d}x^2}+\left(1-\frac{\theta_0}{2}\right)\left(\frac{\mathrm{d}\theta_0}{\mathrm{d}x}\right)^2\right]\right\} = a\theta_0+b\theta_0^3 \tag{35}$$

with

$$a=2(\xi_2^2q_y^2+\xi_3^2q_z^2+\cos 2\alpha) \tag{36 a}$$

or, using equation (16) with $q_x=0$

$$a=2(\cos 2\alpha-\cos 2\alpha_{\mathrm{c}}) \tag{36 b}$$

and

$$b=\xi_3^2q_z^2\left(\frac{K_1}{K_3}-1\right)-\cos 2\alpha. \tag{37}$$

In the $K_1=K_3$ approximation, equation (35) can be easily solved by the same method followed for equation (24), giving the solution

$$\theta_0(x)=\theta_0\,\mathrm{th}\left[\frac{(-a)^{1/2}}{2\xi_1}x\right] \qquad (a<0) \tag{38}$$

where θ_0 is given by (28) with (36–37), now with $K_1=K_3$. This solution corresponds to a splay–bend wall, of width of order

$$l_{\mathrm{sb}}\approx 2\xi_1/(-a)^{1/2} \tag{39}$$

which, as in the twist wall case, shows a divergence at the transition point ($a=0$).

The energy per unit surface stored in the splay–bend wall can be computed, with the same method used for the preceding case, in the $K_1=K_3\equiv K$ approximation, giving

$$W_{\mathrm{sb}}=\frac{1}{3}\frac{K}{\xi}\frac{(-a)^{3/2}}{(-\cos 2\alpha)}, \qquad \xi=\left(\frac{K}{\chi_{\mathrm{a}}H^2}\right)^{1/2} \qquad (a<0) \tag{40}$$

which is positive, since for $a<0$, $\alpha>\alpha_{\mathrm{c}}\geqslant 45°$.

Finally, a bend–splay wall is sought with the ansatz

$$\theta(\mathbf{r})=\theta_0(z)\sin\Omega, \qquad \Omega=q_xx+q_yy. \tag{41}$$

Following the same method as in the preceding case, we get for such a wall

$$\theta_0(z)=\theta_0\,\mathrm{th}\left[\frac{(-a)^{1/2}}{2\xi_3}z\right] \qquad (a<0) \tag{42}$$

with

$$a=2(\xi_1^2q_x^2+\xi_2^2q_y^2+\cos 2\alpha) \tag{43 a}$$

or, using (16) with $q_z=0$

$$a=2(\cos 2\alpha-\cos 2\alpha_{\mathrm{c}}) \tag{43 b}$$

of width of order

$$l_{\mathrm{bs}}\approx 2\xi_3/(-a)^{1/2} \tag{44}$$

which shows the same critical behaviour as in the preceding cases.

The energy per unit surface stored in the bend–splay wall can be computed, in the $K_1=K_3$ approximation, with the same method used in the preceding case. The result is again expression (40), now with a given by (43). This expression, as well as expression (32) in the $K_1=K_3$ case, appears to diverge at $\alpha=45°$. This deserves the following comment: the formation of a wall occurs at the critical angle (when $a=0$), and since for a sample with $\chi_{\mathrm{a}}>0$ $\alpha_{\mathrm{c}}\in[45°, 90°]$, this means that only when $\alpha_{\mathrm{c}}=45°$ could a divergence arise, but in this case the energy density is

$$W_{\alpha_{\mathrm{c}}=45°}\propto\frac{K}{\xi}(-\cos 2\alpha)^{1/2} \tag{45}$$

which is properly equal to zero when $\alpha=45°$. This argument is readily generalized for expression (32) in the $K_1\neq K_3$ case.

4. Critical exponents

The calculation of the critical exponents, corresponding to the transition at $\alpha=\alpha_{\mathrm{c}}$, in the oblique magnetic field geometry, allows further insight to be obtained on the nature of that transition. The interesting exponents in this case are β, ν and γ.

The exponent β, associated with the order parameter, can be easily obtained. The quantity a given by equations (25 *b*), (36 *b*) or (43 *b*) is a measure of the deviation from the critical angle. From equation (28) we get for the order parameter above the transition

$$\theta_0\propto(-a)^{1/2} \tag{46}$$

giving the classical (mean field) value $\beta=1/2$, which characterizes second order phase transitions in mean field models [9]. This is consistent with the form of the (adimensional) free energy given by equation (29).

The exponent ν, associated with the correlation length, can be obtained from the expressions (30), (39) or (44) for the width of the walls

$$l \propto (-a)^{-1/2} \tag{47}$$

giving $\nu = 1/2$, again the classical value [9]. This divergence of l at the transition point can then be explained in the frame of the general theory of second order transitions, where the correlation length diverges.

In order to obtain the exponent γ, associated with the susceptibility, perturbations to the potential (29) are now considered. This potential applies to a perfect nematic monodomain with free boundaries and is clearly not valid in the presence of defects of the nematic director field. There is a formal way of generalizing the former expression in order to take defects or boundary effects into account, with the help of catastrophe theory. In the frame of catastrophe theory, it can be shown that the most general perturbation of the perfect potential (29) is a linear term in the order parameter θ_0 [7], giving for the imperfect potential

$$\Phi_{\text{imp}} = \Phi_0 + \sigma\theta_0 + a\theta_0^2/2 + b\theta_0^4/4 \tag{48}$$

where σ is called the imperfection parameter. In equation (48), the potential $\Phi_{\text{imp}} - \Phi_0$ has the form of a *cusp catastrophe*. The adding of a linear term to (29) has a symmetry-breaking consequence, as shown in figure 3. The second-order transition will disappear under an arbitrarily small symmetry-breaking perturbation, and the transition may reappear at a distant point as a first order transition [7]. This behaviour is related to the structural instability of second order phase transitions.

An immediate consequence of adding the linear term in the order parameter to the potential (29) is that one can now easily obtain from it the critical exponent γ. From $\partial^2 \Phi_{\text{imp}}/\partial\sigma\partial\theta_0 = 0$ one gets for the equivalent of a susceptibility

$$\left(\frac{\partial\theta_0}{\partial\sigma}\right)_{\sigma=0} \propto a^{-1} \tag{49}$$

giving $\gamma = 1$, again the classical value. In the frame of a phase transition analogy, the parameter σ is then the field conjugate to the order parameter θ_0, similarly to the case of a ferromagnetic system [9]. The critical exponent γ of the problem under study can then be associated with the analogue of the zero-field isothermal susceptibility.

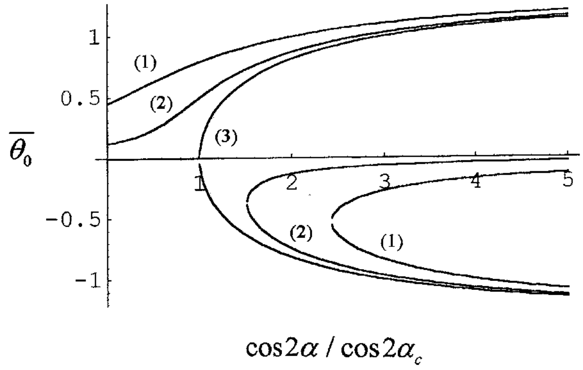

Figure 3. Plot of the equilibrium value θ_0 for the potential (48) as a function of $\cos 2\alpha/\cos 2\alpha_c$, with a given by equation (25) and b by (26), with $q_x = 0$ and $K_3/K_1 = 0{\cdot}5$, for three values of the imperfection parameter: (1) $\sigma = 1$, (2) $\sigma = 0{\cdot}25$, (3) $\sigma = 0$. This plot shows the symmetry-breaking effect of a non-zero σ. Curve (3) gives the equilibrium value (28) of the perfect potential (29), which is the bifurcation diagram in the form of the standard trident characteristic of a second order phase transition.

5. Conclusions

The state of the director of a nematic in an oblique magnetic field is determined by the equilibrium points of the potential that describe the system. A stability analysis shows that a bifurcation occurs at the first critical point of the potential. This determines the transition, at a critical angle $\alpha = \alpha_c$ of the magnetic field with the initial director, from the homogeneous to the deformed state.

In the oblique magnetic field geometry, above the transition at α_c, two equally stable configurations are allowed. It was shown that there can be formed twist, splay–bend or bend–splay walls between the corresponding domains, of well defined energy. The width of the walls diverges as the transition is approached from above.

Finally, the calculation of the critical exponents corresponding to the transition and wall formation at $\alpha = \alpha_c$ allows the transition to be understood as a mean field second order phase transition.

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